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LETTER

GLS strength prediction of glass-fiber-reinforced polypropylene

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Discontinuous-glass-fiber-reinforced plastic (GFRP) composite material is widely used in industrial fields, mainly because glass fiber improves the strength and stiffness of polymer and because it is much less expensive than carbon fiber. It is thought that the use of long fiber is important in more efficiently improving the strength and stiffness of composites. In our previous study [1], we reported that the fracture mode of the discontinuous-fiber-reinforced composite changes from the matrix-cracking mode to the fiber-breaking mode (Fig. 1), when the aspect ratio for the fiber length to the fiber radius exceeds about 150. We also demonstrated that the strength is dramatically improved compared to the strength of the short fiber-reinforced composites and that the global load-sharing (GLS) model can roughly predict the strength (Fig. 2). Recently, Thomason [2, 3] produced the long discontinuous-fiber-reinforced composites where the aspect ratio was about 250 and reported the stiffness and strength of the composites. In this article, we applied the GLS model to his experiments and discuss the validity of our models.

First, we discuss the strength of unidirectional (UD) discontinuous-fiber-reinforced composites, σ_{UD}, based on the GLS assumption [4, 5]. We applied two types of GLS approaches to predict the composite strength. The GLS model focuses on one fragmented fiber (i.e., discontinuous fibers), aligned in the fiber axial direction, and neglects the interaction among fibers in the fiber cross-sectional direction. It predicts the composite's strength by simulating the fiber damage evolution in such a fiber. One approach is based on Monte Carlo simulation [6] for fragmentation in a fiber in the composites. The other is based on the analytical model by Duva et al. [5]. (Hereafter, we refer to this as the DCW model.) Monte Carlo simulation deals with a detailed fiber stress distribution and fragment distribution, though multiple calculations are required for the prediction because it is a probabilistic approach. In contrast, the DCW model assumes an approximate stress distribution and fragment distribution, but it predicts the composite strength analytically.

In simulating the fiber-damage evolution, the first approach utilized Monte Carlo simulation with the elastic–plastic hardening shear-lag model given by Okabe and Takeda [7]. The schematic of the elastic–plastic shear-lag model is illustrated in Fig. 3. The axial length of the model was set to $25 \times l_{\mathrm{f}}$ (l_{f} is the length of discontinuous fiber), and the axial length was divided into 10,000 segments. The fiber ends in discontinuous-fiber-reinforced composites were represented by setting some random segments to the initially broken segments. Thus, the averaged length l_{f} of the discontinuous fibers was related to the density of the initially broken segments introduced in the model. The transverse length of the matrix shear region in the model was set to $D = \left(\sqrt{2\pi/\sqrt{3}V_{\mathrm{f}}} - 2\right) r_{\mathrm{f}}$ (r_{f} is the fiber radius, and V_{f} is the fiber volume fraction). The matrix plasticity was introduced by the linear-isotropic hardening function $\bar{\sigma} = \sigma_{\mathrm{y}} + F_{\mathrm{m}}\bar{\varepsilon}^{p}$, where $\bar{\sigma}$ is the effective stress, $\bar{\varepsilon}^{p}$ is the equivalent plastic strain, σ_{y} is the matrix yield stress, and F_{m} is the matrix plastic modulus. With this model, we can calculate the fiber axial-stress distribution.

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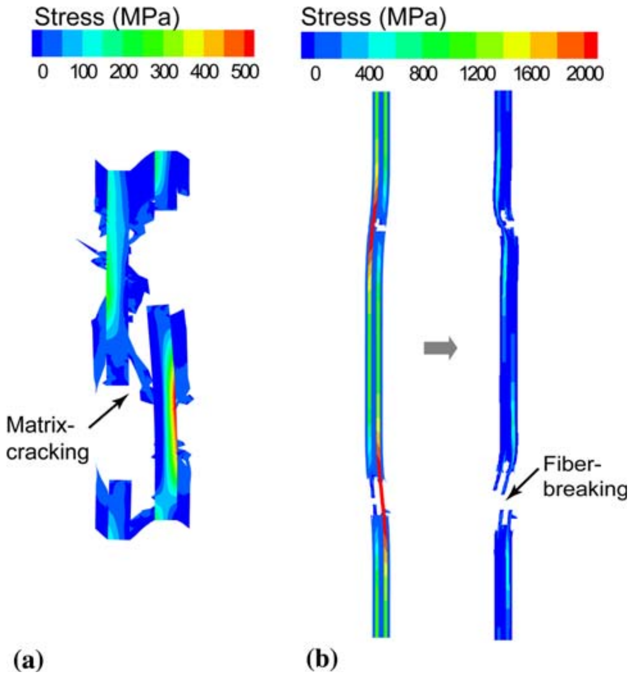

Fig. 1 Microscopic damage transition in discontinuous-fiber-reinforced plastics. The simulated results are extracted from Ref. [1]. **a** Fiber length: 0.1 mm (aspect ratio: 15.4), **b** fiber length: 2.0 mm (aspect ratio: 308)

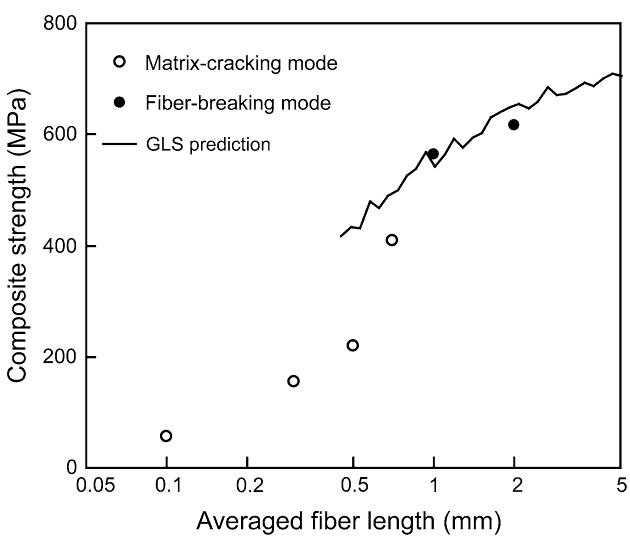

Fig. 2 Composite strength versus fiber length in discontinuous-glass-fiber-reinforced polypropylene. The data was extracted from Ref. [1]

This stress analysis was then incorporated into a Monte Carlo simulation [6] to address the fiber-damage evolution. We first assigned the fiber strengths to initially unbroken segments based on the Weibull model. In the Weibull model, the cumulative failure probability P^{f} of a fiber segment of length Δ is given as

$$P^{\mathrm{f}} = 1 - \exp\left\{-\frac{\Delta}{L_0}\left(\frac{\sigma}{\sigma_0}\right)^{\rho}\right\} \tag{1}$$

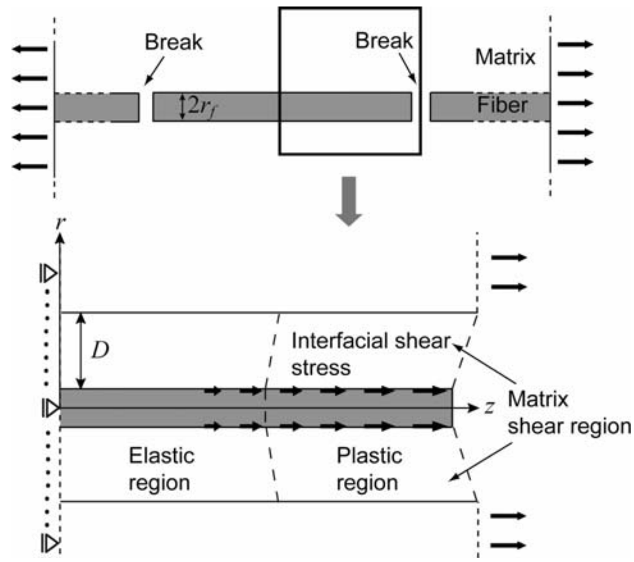

Fig. 3 Schematic of elastic–plastic shear-lag model

where ρ is the Weibull modulus, and σ_0 is the characteristic strength of the fiber with length L_0. The actual strength for each fiber segment was calculated by substituting a random number within (0, 1) into P^{f} in Eq. 1. We judged the segments to be broken when the stress reached the assigned strength with increasing applied strain. In short, we conducted a Monte Carlo simulation for the fiber-damage evolution in unidirectional discontinuous-fiber-reinforced composites. The fiber bundle stress σ_{b} of UD composites was then determined as a function of the applied strain by summing the fiber axial stress and averaging it for all segments.

The second approach is based on the GLS model given by Duva et al. [5]. Following the DCW model, the cumulative number of breaks N in a fiber with length L subjected to far-field fiber stress σ_{f} is given to deal with discontinuous fibers as

$$N = L\left\{d_0 + \frac{1}{L_0}\left(\frac{\sigma_{\mathrm{f}}}{\sigma_0}\right)^{\rho}\right\} \tag{2}$$

where d_0 is the initial density of discontinuities in a fiber. σ_{f} can be calculated by multiplying the fiber Young's modulus E_{f} and the applied strain. The density d_{b} of breaks is given by N/L. Considering the fragment distribution in such a fiber, Duva et al. demonstrated that the fiber bundle stress σ_{b} of a UD composite is given as

$$\sigma_{\mathrm{b}} = \frac{\sigma_{\mathrm{f}}}{\Phi}(1 - \exp(-\Phi)) \tag{3}$$

where $\Phi = 2d_{\mathrm{b}}L_T = 2L_T\left\{d_0 + \frac{1}{L_0}\left(\frac{\sigma_{\mathrm{f}}}{\sigma_0}\right)^{\rho}\right\}$. Here L_T is the stress recovery length from a break. We substituted the solution of elastic–plastic hardening shear-lag model into L_T and then we can calculate the stress–strain response in relation to the cumulative fiber breaks.

With these two models, we calculated the stress–strain response until the fiber bundle stress dropped to 90% of the maximum attained stress. Since the fiber bundle strength $\sigma_{b,cr}$ of a UD composite can be determined as that maximum stress, we can predict the strength of a unidirectional discontinuous-fiber-reinforced composite as follows:

$$\sigma_{UD} = V_f \sigma_{b,cr} + (1 - V_f)\sigma_m \quad (4)$$

where σ_m is the tensile matrix stress. The strength σ_{comp} of the actual discontinuous-fiber-reinforced composite is modified considering the fiber orientation as follows:

$$\sigma_{comp} = \eta V_f \sigma_{b,cr} + (1 - V_f)\sigma_m \quad (5)$$

where η is the orientation factor.

With this procedure, we predicted the strength of long discontinuous-fiber-reinforced composites and compared it to the experiment value reported by Thomason [2, 3]. In this study, we used $r_f = 10$ µm and $l_f = 3.11 - 0.03w_f$ (mm) referring to his experiment. w_f is the weight fraction of fiber and $w_f = 73\%$ corresponds to $V_f = 0.5$ [3]. $\eta = 0.532 \pm 0.001\ w_f$ was used as the orientation factor predicted with the optical method in that study. The material and strength properties used in this calculation are listed in Table 1. These material properties were the same as those used in our previous calculation for glass-fiber-reinforced polypropylene [1], except that the matrix yield stress was adjusted by fitting with the tensile stress–strain data of a polypropylene matrix given by Refs. [2, 3]. The detailed strength properties of glass fiber were taken from the values obtained in our previous experiments [8]. We preliminarily investigated the effect of fiber length distribution on the predicted results, comparing constant length distribution and experimentally measured length distribution reported in Ref. [3]. For example, for $w_f = 19\%$, the predicted strengths obtained with Monte Carlo simulation were 120 MPa for constant distribution and 124 MPa for measured distribution. From these results, we consider that the effect of length distribution is not significant for this material system. Thus in the following Monte Carlo simulation results, we inputted the constant fiber-length distribution.

Table 1 Material properties used in the predictions

Fiber Young's modulus, E_f	76 GPa
Fiber strength σ_0 based on $L_0 = 24$ mm	1550 MPa
Weibull modulus, ρ	6.34
Matrix Young's modulus, E_m	1.8 GPa
Matrix Poisson's ratio, ν_m	0.33
Matrix yield stress, σ_y	26 MPa
Matrix plastic modulus, F_m	10 MPa

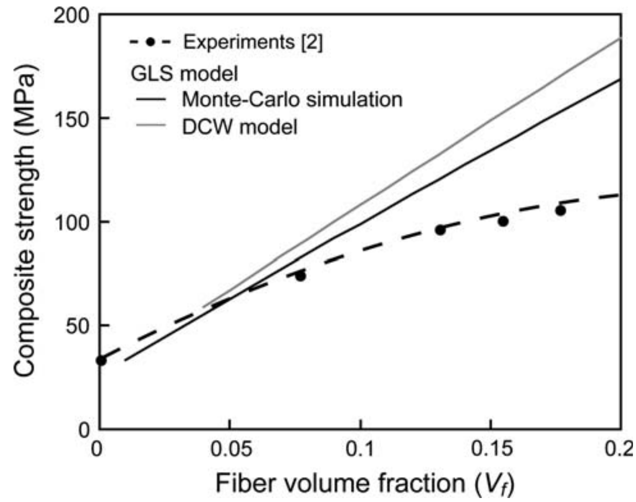

Fig. 4 Composite strengths predicted with GLS model as the fiber content is varied

Figure 4 presented the simulated results as the fiber content (V_f) is varied. When the fiber content is small, the predicted results agreed well with the experiments. For example, for $V_f = 0.08$, the predicted composite strength with the Monte Carlo simulation was 84 MPa, and that with the modified DCW model was 92 MPa. The predicted strengths were close to the experimental value (≈70–75 MPa). However, as the fiber content increases, the predicted strength is much higher than the experimental value. As implied by Thomason [3], within small fiber content, the final failure is controlled by fiber-breakage mode. Therefore, the inconsistency between the prediction and experiment implies that the matrix-cracking mode occurs as the fiber content increases. The limitation of the GLS model focusing on the fiber-breakage mode will be discussed in our future study.

Through this comparison, we could draw two important conclusions. First, the GLS model can predict the experimental values of the long discontinuous-fiber-reinforced composites. The applicability of the GLS model to the long discontinuous-fiber-reinforced composites has never been reported before. In particular, the DCW model works very well as an approximate model. Second, this agreement implies that the fracture mode of the discontinuous-fiber-reinforced composite changes from the matrix-cracking mode to the fiber-breaking mode when the aspect ratio for the fiber length to the fiber radius is sufficiently large. Generally, it is difficult to handle a long fiber in injection molding, which is usually used as the forming method. In addition, manufacturing cost increases with the fiber length. Therefore, it is important to choose the appropriate fiber length in order to balance cost and performance. In this article, we confirmed that the fracture mode changes when the aspect ratio for the fiber length is about 250.

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